1. Determine whether the table represents a discrete probability distribution.

х	P(x)
4	0.35
5	0.2
6	0.2
7	0.25

Answer: Yes, it is. The sum of the probabilities: 0.35 + 0.20 + 0.20 + 0.25 = 1

2. The following distribution is *not* a probability distribution because _____.

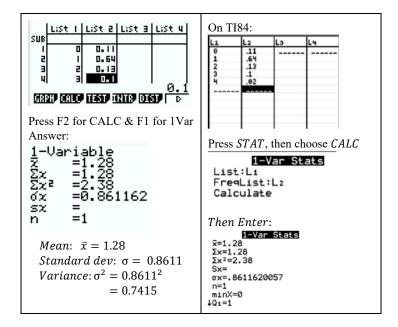
- A) the probability values are not discrete B) values of the variable are negative
- C) the probability values do not add to 1
- D) a probability is negative

3. The following table presents the probability distribution of the number of vacations X taken last year for a randomly chosen family. Find P (1 or more).

$$P(1 \text{ or more}) = P(1) + P(2) + P(3) + P(4) = 0.91$$
 or $P(1 \text{ or more})$ same as $P(\text{at least } 1) = 1 - P(0) = 1 - 0.09 = 0.91$

4. The following table presents the probability distribution of the number of vacations X taken last year for a randomly chosen family. Compute the mean, the variance and the standard deviation of the variable

```
\mu = \sum (x \cdot p(x)) = 0(0.11) + 1(0.64) + 2(0.13) + 3(0.1) + 4(0.02) = 1.28
\sigma^2 = \sum (x^2 \cdot p(x)) - \mu^2 = 0(0.11) + 1(0.64) + 4(0.13) + 9(0.1) + 16(0.02) - 1.28^2 = 2.38 - 1.6384 = 0.7416
Standard deviation, \sigma = \sqrt{Variance} = \sqrt{(0.7416)} = 0.8611
```



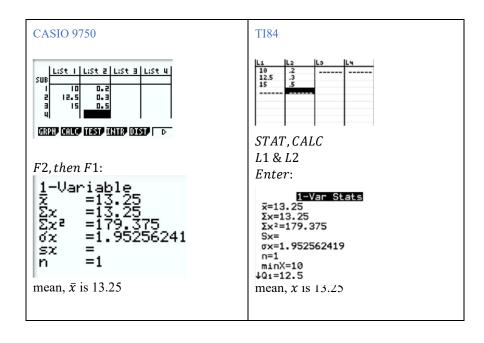
5. A lab orders a shipment of 100 rats a week, 52 weeks a year, from a rat supplier for experiments that the lab conducts. Prices for each weekly shipment of rats follow the distribution below:

Price	\$10.00	\$12.50	\$15.00
Probability	0.2	0.3	0.5

How much should the lab budget for next year's rat orders assuming this distribution does not change. Hint: Find de expected price (or value).

Answer: Find the mean, expected value. The given probability distribution corresponds to *weekly shipment of rats;* but the question asks for *lab budget for next year;* therefore, once the expected value (mean) for a week is found, multiply the result for 52, the number of weeks in a year.

Using the formula, $\mu = \sum (x \cdot p(x)) = 10(0.2) + 12.50(0.3) + 15(0.5) = 13.25$ per week Then, \$13.25(52 weeks) = \$689.00 for the year.



6. A student takes a 15-question, multiple-choice exam with three choices for each question and guesses on each question. Find the probability of guessing exactly 2 out of 15 correctly.

Binomial probability question: we are given the probability of success for one individual trial (1 out of the 3 answer choices; therefore, p of success is 1/3 and probability of failure, q, is 2/3) if the student guesses on every question, 15 in total (n = 15) what is the probability of getting exactly 2 correct (x = 2)?

Binomial formula: $P(x) = nCx \ p^x \cdot q^{n-x}$ substituting values: $P(2) = 15C2(1/3)^2(2/3)^{13} = 0.0599 ... \approx 0.060$

Binomial pdf on Calculators:



7. The Australian sheep dog is a breed renowned for its intelligence and work ethic. It is estimated that 30% of adult Australian sheep dogs weigh 65 pounds or more. A sample of 17 adult dogs is studied. What is the probability that no more than 3 of them weigh 65 lb. or more?

Binomial probability question: p = 30% = 0.30; $\therefore q = 0.7$; n = 17, find x no more than 3 implies $x \le 3$

Binomial formula: $P(x) = nCx \ p^x \cdot q^{n-x}$ since $x \le 3$, we need to find P(0) + P(1) + P(2) + P(3):

 $P(0) = 17C0(0.30)^{0}(0.7)^{17} = 0.0023$

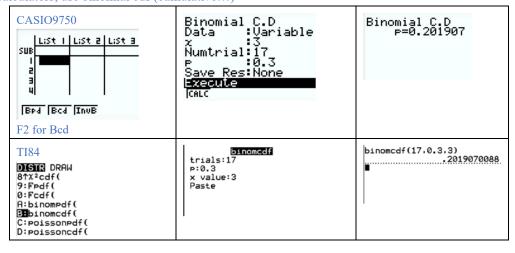
 $P(1) = 17C1(0.30)^{1}(0.7)^{16} = 0.0169$

 $P(2) = 17C2(0.30)^{2}(0.7)^{15} = 0.0581$

 $P(3) = 17C3(0.30)^3(0.7)^{14} = 0.1245$

Total sum = 0.2018

On calculators, use binomial cdf (cumulative...)

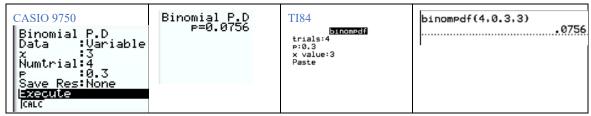


8. In a large bag of marbles, 30% of them are red. A child chooses 4 marbles from this bag. If the child chooses the marbles at random, what is the chance that the child gets exactly three red marbles?

Binomial probability question: probability of success, 30% or 0.30 decimal. The number of trials, n = 4. What is P(x = 3)?

```
Formula: P(x) = nCx \ p^x \cdot q^{n-x} substituting values: P(3) = 4C3(0.3)^3(0.7)^1 = 0.0756
```

On Calculators:



9. A jewelry supplier has a supply of earrings which are 30% platinum. A store owner orders five sets of earrings from the supplier. If the supplier selects the pairs of earrings at random, what is the chance that the jewelry store gets exactly two sets of platinum pairs?

Binomial probability question: we are given the probability that 30% or 0.30 of all earrings are made of platinum. If a specific number of them (5) are selected at random, we must determine the probability that two of them are made of platinum (x = 2).

Formula: $P(x) = nCx \ p^x \cdot q^{n-x}$ substituting values: $P(3) = 5C2 \ (0.3)^2 (0.7)^3 = 0.3087$

Binomial on Calculators:



10. It is estimated that 40% of households own a riding lawn mower. A sample of 13 households is studied. What is the probability that more than 10 of these own a riding lawn mower?

```
Binomial probability question: p = 0.40 therefore, q = 0.60; n = 13 What is the probability x > 10?

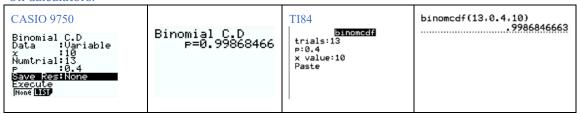
We need to find P(11) + P(12) + P(13)

Formula: P(x) = nCx \ p^x \cdot q^{n-x} substituting values: P(11) = 13C11 \ (0.4)^{11} (0.6)^2 = 0.0012 P(12) = 13C12 \ (0.4)^{12} (0.6)^1 = 0.0001 P(13) = 13C13 \ (0.4)^{13} \ (0.6)^0 = 0.000007 Sum = 0.001307 \approx 0.0013
```

On calculators. The sum of probabilities from x = 0 to x = 13 is x = 1 (total probability); but we need the probability from x = 11 to x = 13. The binomial CDF yields the cumulative probability from x = 0 to a number, x = 0.

Therefore, if we subtract the cumulative probability from zero to ten from 1, the difference is P(x > 10). 1 – P (bin CDF x = 10) = 1 – 0.9987 = 0.0013

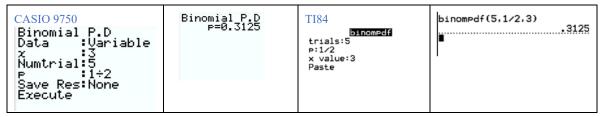
On Calculators:



11. A coin is tossed five times. Find the probability of getting exactly three heads.

```
Binomial probability: n=5, x=3 and probability p=\frac{1}{2}
Formula: P(x)=nCx p^x\cdot q^{n-x} substituting values: P(3)=5C3(1/2)^3(1/2)^2=0.3125\approx 0.313
```

On Calculators:



12. A university has 10,000 students of which 55% are male and 45% are female. If a class of 30 students is chosen at random from the university population, find the mean of the number of male students.

Mean of a binomial distribution: if the probability of males is 0.55, in a group of 30 students chosen at random, how many we expect to be males?

```
As an average: \mu = n \cdot p = 30 \times 0.55 = 16.5
```

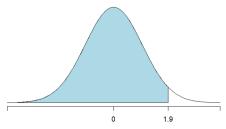
13. In a survey, 65% of the voters support a particular referendum. If 10 voters are chosen at random, find the standard deviation of the number of voters who support the referendum.

```
Standard deviation of a binomial experiment where n=10, p=0.65 & q=1-p=0.35 \sigma=\sqrt{n\cdot p\cdot q}=\sqrt{10\cdot 0.65\cdot 0.35}=1.508\dots
```

14. A coin is tossed 72 times. Find the standard deviation for the number of heads that will be tossed.

```
Again, standard deviation of a binomial experiment where n=72, p=0.5 & q=1-p=0.5 \sigma=\sqrt{n\cdot p\cdot q}=\sqrt{72\cdot 0.5\cdot 0.5}=4.2426...
```

15. Find the area under the standard normal curve to the left of z = 1.9

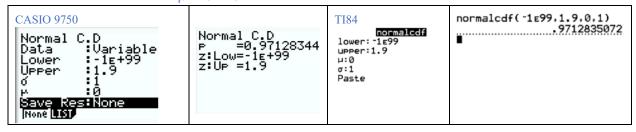


Z score, this is about the normal distribution: the standard normal distribution, z-scores, for which $\mu = 0$, $\sigma = 1$. Z -score tables show the area to the left of a given z -score. All we need is to locate z = 1.9 on the table.

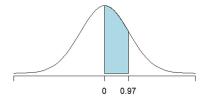
P(z < 1.9) = 0.9713

1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

On Calculators for a z-score $\mu = 0$ $\sigma = 1$



16. The probability P(0 < z < 0.97) is 0.3340



Area to the left of z = 0.97 or P(z < 0.97) = 0.8340

(0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
().9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1	.О	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1	.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1	.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015

Area to the left of z = 0.0 or P(z < 0.00) = 0.5000

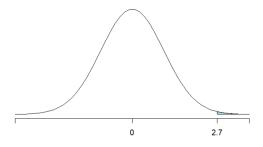
TABLE A-2 (continued) Cumulative Area from the LEFT													
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09			
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359			
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753			

Area in between given by: 0.8340 - 0.5000 = 0.3340

On Calculators. Since these are z-scores, the mean, $\mu = 0$ and the Standard deviation, $\sigma = 1$.



17. Find the area under the standard normal curve to the right of z = 2.7

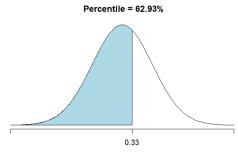


Area to the right of z = 2.7 or P(z > 2.70) = 1 - P(z < 2.7) = 1 - 0.9965 = 0.0035

2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

CASIO 9750 Normal C.D Data :Variable Lower :2.7 Upper :1e+99 of :1 p :0 Save Res:None None List	Normal C.D P = 3.467e-03 z:Low=2.7 z:UP =1e+99	normalcdf lower:2.70 upper:£99 µ:0 σ:1 Paste	normalcdf(2.70, e99,0.1) .0034670231
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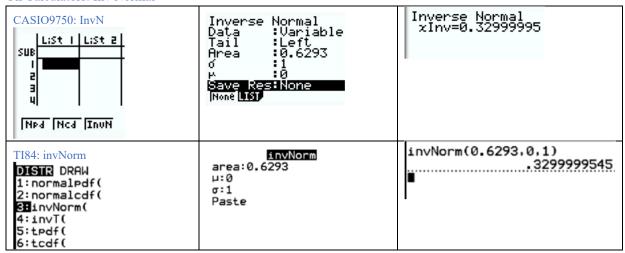
18. Find the z value to the right of the mean so that 62.93% of the area under the distribution curve lies to the left of it.



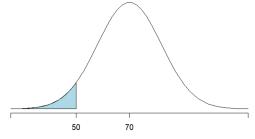
62.93% as decimal = 0.6293. What is the z-score that corresponds to the given probability or area? Z = 0.33

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224

On Calculators: Inv Normal



19. If a normally distributed group of test scores have a mean of 70 and a standard deviation of 12, find the percentage of scores that will fall below 50.



Normal distribution question, actual values given (not z-scores), n=1. Find the z-score in order to use the z-score table if you don't have a Graphing calculator: P(x < 50); $\mu = 70$ and $\sigma = 12$

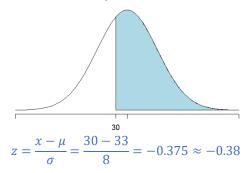
$$z = \frac{x - \mu}{\sigma} = \frac{50 - 70}{12} = -1.6666 \dots \approx -1.67$$

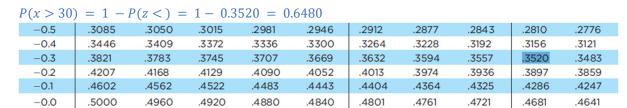
P(z < 50) = P(z < -1.67) by table = 0.0475 as a percentage 0.0475 $\times 100 = 4.75\%$

-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	.0465	.0455

CASIO 9750 Normal C.D Data Variable Lower -1E+99 UPPER 50 0 12	Normal C.D P =0.04779035 z:Low=-8.333E+97 z:UP =-1.6666667	TI84 normalcdf lower: -1E99 upper:50 u:70 o:12 Paste	normalcdf(-1E99,50,70,12) .0477903304
O : 14 P : 70 Save Res: None Rone LIST		Paste	

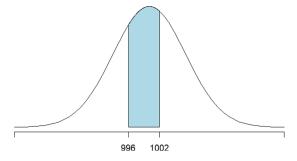
20. A normal population has a mean $\mu = 33$ and standard deviation $\sigma = 8$. What is the probability that a randomly chosen value will be greater than 30?





CASIO 9750		TI84	normalcdf(30, E99, 33, 8)
Normal C.D Data :Variable Lower :30 Upper :1e+99 6 8 8 8 8 33 Save Res:None	Normal C.D P =0.64616976 z:Low=-0.375 z:UP =1.25e+98	normalcdf lower:30 upper:e99 µ:33 σ:8 Paste	.6461697127

21. A bottler of drinking water fills plastic bottles with a mean volume of 1000 milliliters (mL) and standard deviation 7 mL. The fill volumes are normally distributed. What is the probability that a bottle has a volume between 996 mL and 1002 mL?



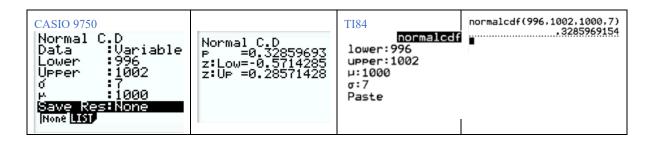
Find both z-scores, for x = 1002

$$z = \frac{x - \mu}{\sigma} = \frac{1002 - 1000}{7} = 0.2857 \dots \approx 0.29$$

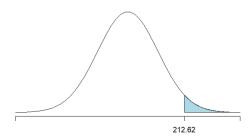
U	/									
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	5000	F0.40	F000	F100	F100	F100	5070	5070	F.710	F750
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

For x = 996:

Probability of area or probability in between these two values is given by the difference of the areas: 0.6141 - 0.2843 = 0.3298



22. Mrs. Smith's reading class can read an average of 175 words per minute with a standard deviation of 20 words per minute. The top 3% of the class is to receive a special award. What is the minimum number of words per minute a student would need to read in order to get the award? Assume the data is normally distributed.



 $\mu = 175$, $\sigma = 20$, which is the x value that separate the top 3% of the class from the bottom 97%? Firstly, find the z –score that corresponds to 0.9700 (remember that z-score tables list the left or bottom part of the distribution:

1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

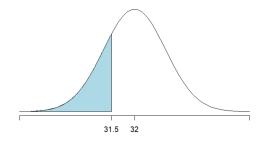
The z –score that corresponds to 0.9700 is 1.88

Solving from the z-score table, $x = \mu + z \cdot \sigma = 175 + 1.88 (20) = 212.6 \approx 213$

On Calculators, use Inv Normal:



23. A survey of 250 lobster fishermen found that they catch an average of 32.0 pounds of lobster per day with a standard deviation of 4.0 pounds. If a random sample of 36 lobster fishermen is selected, what is the probability that their average catch is less than 31.5 pounds? Assume the distribution of the weights of lobster is normal.



 $\mu = 32.0$ $\sigma = 4.0$ n = 36 $P(\bar{x} < 31.5)$ Notice that sample size is other than one; therefore, we must use the z-score formula based on the Central Limit Theorem: finding z-score:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{31.5 - 32.0}{4/\sqrt{36}} = -0.75$$
 Therefore, $P(\bar{x} < 31.5) = P(z < -0.75) = 0.2266$ as percentage = 22.66%

P(z < -0.75):

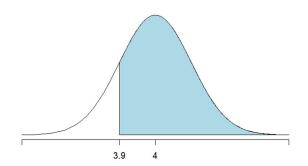
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451

CASIO 9750 Normal C.D Data :Variable Lower : -1E+99 Upper :31.5 0 :4÷√36	normalcdf(-E99,31.5,32,4/) normalcdf(-E99,31.5,32,4/) lower: -E99 upper:31.5 µ:32 σ:4/√(36) Paste
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24. The average diameter of sand dollars (see fig 1A below) on a certain island is 4.00 centimeters with a standard deviation of 0.60 centimeters. If 36 sand dollars are chosen at random for a collection, find the probability that the average diameter of those sand dollars is more than 3.90 centimeters. Assume that the variable is normally distributed.

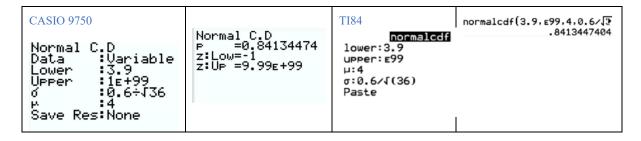


Fig 1A: Sand dollar

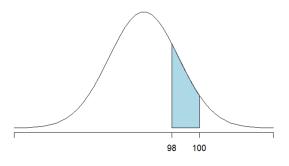


$$\mu = 4.00$$
 $\sigma = 0.60$ $n = 36$ Find $P(\bar{x} > 3.90)$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{3.90 - 4.0}{0.6 / \sqrt{36}} = -1 \qquad P(\bar{x} > 3.90) = P(z > 1) = 1 - P(z < -1) = 1 - 0.1587 = 0.8413$$



25. The average age of vehicles registered in the United States is 96 months. Assume the population is normally distributed with a standard deviation of 15 months. Find the probability that the mean age of a sample of 36 vehicles is between 98 and 100 months?



$$\mu = 96$$
 $\sigma = 15$

$$n = 36$$

$$P(98 < \bar{x} < 100)$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{100 - 96}{15 / \sqrt{36}} = 1.6$$

P(z < 1.6) = 0.9452

1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{98 - 96}{15/\sqrt{36}} = 0.8$$

$$P(z < 0.8) = 0.7881$$

0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

 $P(98 < \bar{x} < 100) = P(0.8 < z < 1.6) = 0.9452 - 0.7881 = 0.1571 \Rightarrow 15.71\%$

On Calculators:

CASIO 9750 Normal C.D Data :Variable Lower :98 Upper :100 0 :15÷√36 p :96 Save Res:None	Normal C.D P =0.1570561 z:Low=0.8 z:UP =1.6	normalcdf lower:98 upper:100 µ:96 σ:15/√(36) Paste	normalcdf(98,100,96,15/\30 .1570560443
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